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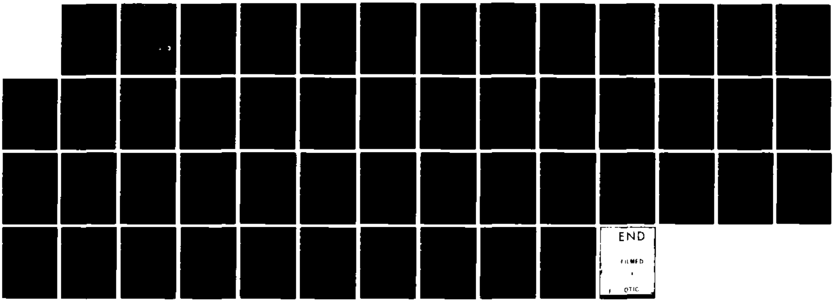

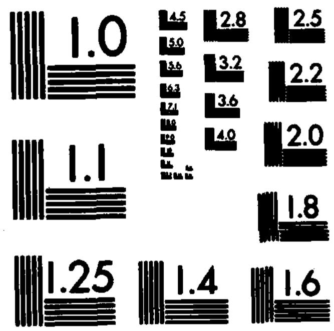

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Algorithms have been developed which minimize the forced vibrational response of structural systems.

The constraints which can be used are displacements or accelerations and natural frequencies. The design variables are linear changes to mass,

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) stiffness or damping matrices. The constraints can be expressed in either the time or frequency domain and the cumulative constraint is used to measure the amount of constraint violation.

It is shown that the variation of the displacement or acceleration constraints are shallow in reciprocal design variables. The objective function is a design variable which represents a value that keeps either displacements or accelerations less than a specified maximum value.

A sequence of linear programs is formulated to take advantage of the almost linear character of the vibration isolation problem.

The algorithms used in the displacement or acceleration constraints are general and consistent with algorithms that have been developed for weight minimizations, but have not been used for this purpose in the study.

These algorithms have been studied for transient response, frequency response and stationary random. The results shown for the minimization of vibration response are very robust. Only passive vibration isolation has been studied and no attempt was made to consider ill conditioned problems.

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OPTIMIZATION FOR VIBRATION ISOLATION

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<u>CONTENTS</u>

1. Introduction

This research considers only passive vibration isolation. Most physical structures designed for dynamic environments have isolator elements to attenuate the response. Examples of problems that could benefit from this research are ground vehicle response and equipment or instrument vibrational response. These problems use passive isolators to minimize the vibrational response.

As a preliminary step toward the study of active structural vibration, the passive vibration is studied in this research. Most Aerospace structures of the future will use active vibration isolation to attenuate the response. The active vibration isolation problem would contain the same constraints used in this study with the addition of control forces to the state equations and to the objective function - Large space structures would use active vibration isolation to control the response of the very flexible structural systems.

The algorithms that are developed in this research are consistent with the minimum weight problem which is not considered in the present study.

This research considers constraints that are displacements, accelerations and natural frequencies. The design variables are linear changes to mass, stiffness or damping matrices. The constraints can be expressed in either the time or frequency domain and the cumulative constraint is used to measure the amount of constraint violation. It is shown that the variation of the displacements or acceleration constraints are shallow in reciprocal design variables. The objective function represents a design variable that restrains displacements or accelerations to be less than a maximum value.

These algorithms have been studied for transient response, frequency response and stationary random. No attempt was made to consider ill conditioned vibrational problems that occur between a structure and isolator elements that are designed to move independent of the structure.

2. Transient Response

The minimization of displacements or accelerations can be formulated as a MIN-MAX optimization problem.

(1) $\mathrm{MIN}\ (\mathrm{MAX}\ |\ddot{X}|)$

(2) $KY = \lambda MY$

(3) $M\ddot{X} + C\dot{X} + KX = P$

(4) $\mathrm{MAX}\ |X_i\ (t) - X_i(t)| \leq X_u$

(5) $\lambda_L \leq \lambda \leq \lambda_U$

(6) $K = K_o + \Sigma\alpha_i K_i$

(7) $M = M_o + \Sigma\alpha_i M_i$

(8) $C = C_o + \Sigma\alpha_i C_i$

(9) $\alpha_{iL} \leq \alpha_i \leq \alpha_{iU}$

Only the direct method of solution has been considered in this study. These algorithms can be used with the modal formulation and a seperate section references the recent work completed using modal analysis.

Equation (1) minimizes the maximum acceleration in the time domain. The objective function could be displacements instead and the present algorithms could be used directly. Equation (2) is the eigenvalue problem whose inclusion permits the use of frequencies in equation (5) in the analysis. Equation (3) is the structural dynamic equations in matrix form which describe the displacement response X(t). Equation (4) is the so called relative displacement or rattlespace constraint. The present algorithms can include this type of constraint in the analysis. However, no specific numerical examples are presented using the rattlespace constraint. Equations (6), (7), and (8) show the linear changes to the stiffness, mass or viscous damping matrix with the design variables α_i. The design variables could contain differing sets in equations (6), (7), (8). Equation (9) lists the constraint limits on the design variables α_i.

3. Frequency Response

Sometimes, it is convenient to solve vibration problems in the driving frequency ω domain. This is true for problems which have experimentally available results for transfer functions. Also, for stationary random analysis, the frequency domain transfer function must be determined. Equation (3) is transformed to the steady state frequency domain by,

$$(10) \quad X = RE\{X_o e^{i\omega t}\}, \quad P = RE\{P_o e^{i\omega t}\}$$

where RE: denotes real part of

$i = \sqrt{-1}$

ω: driving frequency

X_o: amplitude of harmonic response

P_o: amplitude of harmonic loading

For the harmonic substitution, equation (3) becomes,

$$(11) \quad (-\omega^2 M + i\omega C + K)X_o = P_o$$

The amplitudes X_o, P_o are complex numbers. Equation (11) may be solved repeatedly for X_o given P_o and ω using complex arithmetic. It is more convenient to use the real displacement components in the analysis. The method of reference (1) is used to work with the real and imaginary components of X_o.

$X_o = U - iV$

$$(12) \quad \begin{vmatrix} -\omega^2 M + K & \omega C \\ \omega C & \omega^2 M + K \end{vmatrix} \begin{vmatrix} U \\ V \end{vmatrix} = \begin{vmatrix} P_o \\ 0 \end{vmatrix}$$

The optimization becomes in the frequency domain,

$$(13) \quad \text{MIN} \; (\text{MAX} \; (\omega^2 \sqrt{U^2 + V^2})$$

$$(14) \quad KY = \lambda MY$$

$$(15) \quad \begin{vmatrix} -\omega^2 M + K & \omega C \\ \omega C & \omega^2 M + K \end{vmatrix} \begin{vmatrix} U \\ V \end{vmatrix} = \begin{vmatrix} P_o \\ 0 \end{vmatrix}$$

$$(16) \quad \text{MAX} \; | X_{oi}(\omega) - X_{oj}(\omega) | \leq X_U$$

$$(17) \quad \lambda_L \leq \lambda \leq \lambda_U$$

$$(18) \quad K = K_o + \Sigma \alpha_i K_i$$

$$(19) \quad M = M_o + \Sigma \alpha_i M_i$$

$$(20) \quad C = C_o + \Sigma \alpha_i C_i$$

$$(21) \quad \alpha_{iL} \leq \alpha_i \leq \alpha_{iu}$$

Equation (13) is the amplitude of steady state acceleration and equation (15) are the structural dynamic equations to be solved.

4. Stationary Random

A Frequency Response solution is first analyzed to determine the transfer function $H(\omega)$ which is either the displacement or acceleration at a response point of interest. The spectral density of the output is given in terms of the spectral density of the input for a single input/output system is given in reference (2).

$$S_o(\omega) = |H(\omega)|^2 S_I(\omega)$$

The same reference also lists techniques for analyzing multiple input/output systems. The mean square value can be calculated for any frequency interval,

$$\bar{z}^2 = \int_{\omega_1}^{\omega_2} S_o(\omega) \quad \omega.$$

Various performance measures have been proposed for random analysis such as using either the spectral density or mean square value. The optimization problem for stationary random becomes,

(22) $\text{MIN } (\text{MAX } S_o)$

(23) $KY = \lambda MY$

(24) $\begin{vmatrix} -\omega^2 M + K & \omega C \\ \omega C & \omega^2 M + K \end{vmatrix} \begin{vmatrix} U \\ V \end{vmatrix} = \begin{vmatrix} P_o \\ 0 \end{vmatrix}$

(25) $S_o = |H(\omega)|^2 S_I$

(26) $\lambda_{{}_{\mathrm{r}}} \leq \lambda \quad {}_U$

(27) $K = K_o + \Sigma\alpha_i K_i$

(28) $M = M_o + \Sigma\alpha_i M_i$

(29) $C = C_o + \Sigma\alpha_i C_i$

(30) $\alpha_{iL} \leq \alpha_i \leq \alpha_{iU}$

The maximum displacement or acceleration spectral density is the objective function to be minimized in equation (22). Only the single input/output case is used to calculate the spectral density by equation (25), the objective function is converted to a set of equivalent integral constraints and the minimization is done then on the mean square response in effect.

5. Cumulative Constraint and MIN-MAX Problem

The cumulative or equivalent integral constraint has been used in the optimal control literature (3) to convert many discrete points in the time domain to one equivalent integral. Thus many discrete constraint equations are lumped into one equation. The cumulative constraint measures the total amount of constraint violation. If a satisfied constraint is of the form,

(31) $\phi(t) \leq 0$ all t.

The brac function measures the amount of constraint violation,

$$<\phi(t)> = \begin{cases} \phi(t), & \phi(t) \geq 0 \\ 0, & \phi(t) < 0 \end{cases}$$

A constraint totally equivalent to (31) is,

(32) $$\int <\phi(t)>\, dt = 0$$

Equation (32) is not identically zero if the constraint (31) is violated. Instead of using (31) at many discrete points in time, one total constraint (32) is used which measures where the constraint (31) is violated.

The objective function (1), (13) or (22) can be converted to a simplier algebraic form.
Consider equation (1),

$$\text{MIN } (\text{MAX } |\ddot{X}(t)|\ )$$

This minimization is equivalent to minimizing an additional design variable α such that

$$\text{MIN} \quad \alpha$$

$$|\ddot{X}(t)| - \alpha \leq 0 \quad \text{for all } t$$

The acceleration constraint that was just introduced can be converted to a cummlative constraint.

The MIN-MAX part of the optimization becomes,

$$\text{MIN} \quad \alpha$$

(33) $$\int < |\ddot{X}(t)| - \alpha > dt = 0$$

To account for the absolute value of acceleration, the integral is written

$$\int I \, dt = 0$$

$$I = \ddot{X} - \alpha, \quad \ddot{X} \geq \alpha$$

$$I = -\ddot{X} - \alpha, \quad -\ddot{X} \geq \alpha$$

$$I = 0 \qquad \text{otherwise}$$

The objective functions (13) or (22) in the frequency domain are computed in the same manner with frequency replacing time in the integral.

The inner problem or the maximization in this research was done by function evaluation. This is efficient for the transient problem, but the frequency response problem requires a decomposition for each driving frequency in equation (15). It would be required to reduce the basis of equation (15) by using the real normal modes for efficient solution in locating the maximum. Reference (1) recommends performing a one dimensional search on the variable ω to locate the maximum. This one dimensional search would require several initial starting points to insure convergence to the maximum of the nonlinear problem in ω.

6. Reciprocal Design Variables

For statically determinate structures, stresses and deflections are proportional to design variables that are linear changes to stiffness such as areas of rods in truss members. For indeterminate structures, this is only an approximation. It was investigated in references (4,5) and found that high quality explicit expressions for stresses and deflections could be generated using a first order Taylor series expansion in reciprocal design variables. That is, the design variable space for stress and static deflection is shallow in reciprocal design variables. The linearized Taylor series expansions represent lines that are very good approximations to the exact constraints. The expansion of a response quantity ϕ is done in the reciprocal design variables β_i,

(34) $$\phi = \phi_o + \Sigma \frac{\partial \phi}{\partial \beta_i} \delta \beta_i$$

(35) $$\beta_i = \frac{1}{\alpha_i}$$

(36) $$\frac{\partial \phi}{\partial \beta_i} = - \alpha_i^2 \frac{\partial \phi}{\partial \alpha_i}$$

The mass, stiffness and damping matrices are linear functions of the design variables α_i. The derivative of a general matrix G in the above classification is,

(37) $$G = G_o + \Sigma \alpha_i G_i$$

(38) $$\frac{\partial G}{\partial \alpha_i} = G_i$$

(39) $$\frac{\partial G}{\partial \beta_i} = - \alpha_i^2 G_i$$

The derivative can be calculated by setting the linear property equal to one in the element matrix G_i and multiply by the square of the design variable. The same subroutine used to calculate element matrices can be used to form the above derivative.

7. Analytic Derivatives

The direct solution of the dynamic response equations in the time domain uses an efficient implicit equation solver such as Newmark integration. The same technique could be used in the modal formulation if coupled real mode damping matrix exists. This would be the most general capability for solution of the dynamic equations. The Newmark integration equations (6) are listed for one set on integration parameters,

$\delta = \frac{1}{2}$, $\alpha = \frac{1}{4}$,

(40) $$M\ddot{X}_{t_2} + C\dot{X}_{t_2} + KX_{t_2} = P_{t_2}$$

(41) $$KK = K + \frac{4}{(\Delta t)^2} M + \frac{2}{(\Delta t)} C$$

(42) $$PP_{t_2} = P_{t_2} + M\left|\frac{4}{(\Delta t)^2} X_{t_1} + \frac{4}{\Delta t}\dot{X}_{t_1} + \ddot{X}_{t_1}\right| + C\left|\frac{2}{\Delta t}X_{t_1} + \dot{X}_{t_1}\right|$$

(43) $$KK \cdot X_{t_2} = PP_{t_2}$$

(44) $$\ddot{X}_{t_2} = \frac{4}{(\Delta t)^2}\left(X_{t_2} - X_{t_1}\right) - \frac{4}{\Delta t}\dot{X}_{t_1} - \ddot{X}_{t_1},$$

(45) $$\dot{X}_{t_2} = \dot{X}_{t_1} + \frac{\Delta t}{2}\ddot{X}_{t_1} + \frac{\Delta t}{2}\ddot{X}_{t_2}$$

The displacements at the next time step are calculated by equation (43). The matrix KK is only factored when the time step Δt changes. The acceleration and displacements are recovered by equations (44) and

(45). The derivatives of the response quantities (7) are found by differentiating equations (47) THRU (45). This is the pseudo loads technique. The derivatives with respect to the reciprocal variables are,

$$(46) \quad KK\frac{\partial X_{t_2}}{\partial \beta_i} = -\frac{\partial KK}{\partial \beta_i}X_{t_2} + \frac{\partial PP_{t_2}}{\partial \beta_i}$$

$$(47) \quad \frac{\partial KK}{\partial \beta_i} = \frac{\partial K}{\partial \beta_i} + \frac{4}{(\Delta t)^2}\frac{\partial M}{\partial \beta_i} + \frac{2}{\Delta t}\frac{\partial C}{\partial \beta_i}$$

$$(48) \quad \frac{\partial PP_{t2}}{\partial \beta_i} = \frac{\partial P_{t2}}{\partial \beta_i} + M\left|\frac{4}{(\Delta t)^2}\frac{\partial X_{t_1}}{\partial \beta_i} + \frac{4}{\Delta t}\frac{\partial \dot{X}_{t_1}}{\partial \beta_i} + \frac{\partial \ddot{X}_{t_1}}{\partial \beta_i}\right| + C\left|\frac{2}{\Delta t}\frac{\partial X_{t_1}}{\partial \beta_i} + \frac{\partial \dot{X}_{t_1}}{\partial \beta_i}\right|$$

$$(49) \quad \frac{\partial \ddot{X}_{t_2}}{\partial \beta_i} = \frac{4}{(\Delta t)^2}\left|\frac{\partial X_{t_2}}{\partial \beta_i} - \frac{\partial X_{t_1}}{\partial \beta_i}\right| - \frac{4}{\Delta t}\frac{\partial \dot{X}_{t_1}}{\partial \beta_i} - \frac{\partial \ddot{X}_{t_1}}{\partial \beta_i}$$

$$(50) \quad \frac{\partial \dot{X}_{t_2}}{\partial \beta_i} = \frac{\partial \dot{X}_{t_1}}{\partial \beta_i} + \frac{\Delta t}{2}\frac{\partial \ddot{X}_{t_1}}{\partial \beta_i} + \frac{\Delta t}{2}\frac{\partial \ddot{X}_{t_1}}{\partial \beta_i}$$

Using this technique, the derivatives of displacement, velocity and acceleration must be calculated and saved for all degrees of freedon in the finite element model at two neighboring points in time. The KK matrix in (46) was decomposed in the response calculations and would not be factored again in this step.

The pseudo loads technique was applied to the structural equations (24) in the frequency domain.

$$(51)\quad \begin{vmatrix} -\omega^2 M & \omega C \\ \omega C & \omega^2 M - K \end{vmatrix} \begin{vmatrix} \dfrac{\partial U}{\partial \beta_i} \\ - - - \\ \dfrac{\partial V}{\partial \beta_i} \end{vmatrix} = - \begin{vmatrix} -\omega^2 \dfrac{\partial M}{\partial \beta_i} + \dfrac{\partial K}{\partial \beta_i} & \omega \dfrac{\partial C}{\partial \beta_i} \\ \omega \dfrac{\partial C}{\partial \beta_i} & \omega^2 \dfrac{\partial M}{\partial \beta_i} - \dfrac{\partial K}{\partial \beta_i} \end{vmatrix} \begin{vmatrix} U \\ - - \\ V \end{vmatrix}$$

$$+ \begin{vmatrix} \dfrac{\partial Po}{\partial \beta_i} \\ - - \\ 0 \end{vmatrix}$$

As discussed in reference (8) , frequency constraints are inherently nonlinear, Reference (9) suggests if the structure has large fixed masses, direct design variables and a linear Taylor series expansion should be used. To use frequency constraints in the present research, the inverse eigenvalue λ^* should be used,

$$KY = \lambda MY$$

$$\lambda^* KY = MY$$

$$\lambda^* = \frac{1}{\lambda}$$

The justification follows from the Rayleigh quotient:

$$\lambda^* = \frac{Y^T M_o Y + \Sigma\alpha_i Y^T M_i Y}{Y^T K_o Y + \Sigma\alpha_i Y^T K_i Y}$$

where the mass and stiffness are given by equations (6), (7) or (18), (19).

For large fixed masses, M_o dominates the top of the ratio. The reciprocal eigenvalue contains α_i on the bottom. The eigenvectors Y are functions of the design variables, but enter in the same manner at the numerator and denominator. λ^* might be best represented in β_i space.

The derivative is well known and given in reference (10):

$$(52) \quad \frac{\partial\lambda i}{\partial\alpha j} = Y_i^T \left[\frac{\partial K}{\partial\alpha j} - \lambda i \frac{\partial M}{\partial\alpha j} \right] Y_i$$

This derivative has to be expressed in terms of the reciprocal design variables and reciprocal eigenvalue.

$$\lambda_i = \frac{1}{\lambda^*_i} \qquad \beta_j = \frac{1}{\alpha j}$$

$$(53) \qquad \frac{\partial \lambda^* i}{\partial \beta_j} = \frac{\alpha_j^2 \, Y_i^{\,T}}{\lambda_i^2} \left| \frac{\partial K}{\partial \alpha_j} - \lambda_i \frac{\partial M}{\partial \alpha_j} \right| Y_i$$

8. Linearized Constraints

The acceleration cumulative constraint has the following first order Taylor series expansion in the reciprocal variables.

(54)

$$\int_{o}^{T} I \; dt = 0$$

where
$$I = \ddot{X}_o + \Sigma \frac{\partial \ddot{X}}{\partial \beta_i} \delta\beta_i - \beta, \text{ for } \ddot{X} \gtrsim \beta$$

$$I = -X_o - \Sigma \frac{\partial \ddot{X}}{\partial \beta_i} \partial\beta - \beta, \text{ for } -\ddot{X} \gtrsim \beta$$

$$I = 0 \quad \text{For } \ddot{X} \text{ otherwise}$$

This constraint is numerically integrated by a modified trapezoid law which finds those response points above a line for which the constraints are violated. The algorithm interpolates to find the points where the actual constraint is violated.

The steady state acceleration amplitude in the frequency domain is,

$$A = \omega^2 \sqrt{U^2 + V^2}$$

where $X_o = U - iV$ is the amplitude of steady state displacement.

The first order Taylor series expansion for the acceleration amplitude magnitude is,

(55)

$$\int_{o}^{\omega} I \quad d\omega = 0$$

where
$$I = A_o + \frac{\omega^2}{\sqrt{U^2 + V^2}} \quad \Sigma \left(\frac{\partial U}{\partial \beta_i} + \frac{\partial V}{\partial \beta_i}\right) \delta\beta_i - \beta$$

The acceleration spectral density of the output can be expressed in terms of the spectral density of the input and the transfer function for a single input/output system.

$$S_o\ (\omega) = \omega^4 \left| H(\omega) \right|^2 S_I(\omega)$$

Where the transfer function is the displacement at the response point.

To utilize the same approximation as the frequency response acceleration constraint equation (55), consider the square root spectral density.

$$S = \sqrt{S_o\ (\omega)} = \omega^2\ |H\ (\omega)|\ \sqrt{S_I(\omega)}$$

To modify (55) for acceleration spectral densities, the ω^2 is multiplied by $\sqrt{S_I(\omega)}$.

$$(56) \qquad \int_0^{\omega} I\ d\omega = 0$$

where

$$I = S_o + \frac{\omega^2 \sqrt{S_I\ (\omega)}}{\sqrt{U^2 + V^2}} \quad \Sigma\left(\frac{\partial U}{\partial \beta_i} + \frac{\partial V}{\partial \beta_i}\right) \delta\beta_i - \beta$$

A formula could be derived for multiple sources with cross correlation in a similar manner. Equation (56) developed with this algorithm is the square root mean square acceleration that exceeds a specified value in the frequency domain.

9. Modal Method

The direct method was analyzed in this research, but the developed algorithms could be used with the modal analysis method with the addition of the following algorithms.

The structural dynamic equations are first converted to modal amplitudes as the degrees of freedom.

$$X = YZ$$

$$(57) \quad (Y^T MY)\ddot{Z} + (Y^T CY)\dot{Z} + (Y^T KY)Z = Y^T P$$

The generalized mass and stiffness are diagonal, but the modal damping is in general a coupled matrix as the result of being transform by the real eigenvectors Y. When derivatives are calculated for equation (57) it becomes necessary to differentiate the eigenvector. The research presented in reference (11) should be used to efficiently calculate the eigenvector derivative if large changes have been made to the structure. If only isolator changes are made, usually only small changes are made to the structure.

In such a case, the eigenvector derivative would not be needed. A direct modal solution was done for a problem of this type in reference (12). In this problem, the information could be accurately calculated in the Ritz eigenspace of the current design. A direct modal solution could also be used with the present research. The algorithms of the present research should replace the very inefficient one developed in reference (12).

10. Sequential Linear Programming

The problem considered in this research of minimizing a linear design variable subject to constraints on displacements or accelerations in the time or frequency domain is an almost linear problem in reciprocal design space. It is only natural to use sequential linear programming as the optimization algorithm. A primal-dual linear program which is listed in reference (13) was used as the optimizer. Sequential linear programming is described in reference (14). Reference (15) has shown the dynamic response optimization problem to have a disjoint design space. Sequential linear programming (14) is capable of solving such a nonconvex problem.

The design variables (9), (21) are converted to the reciprocal space,

$$\frac{1}{\alpha_{iu}} \leq \beta_i \leq \frac{1}{\alpha_{iL}} \tag{58}$$

The actual design variables were the changes $\delta\beta_i$,

$$\beta_i = \bar{\beta}_i + \delta\beta_i \quad , \quad \bar{\beta}_i = \frac{1}{\bar{\alpha}_i} \tag{59}$$

where $\bar{\beta}_i$ is the current reciprocal design variable.

The design variables $\delta\beta_i$ are not restricted in sign. Linear programming requires the design variables to be non-negative. This requires the design variable to be converted into the difference of two non-negative variables.

$$\delta\beta_i = \delta\beta_i^+ - \delta\beta_i^- \tag{60}$$

$$\delta\beta_i^+ \geq 0, \quad \delta\beta_i^- \geq 0$$

The bounds on $\delta\beta_i$ become,

$$(61) \qquad \frac{1}{\alpha_{iu}} - \frac{1}{\bar{\alpha}_i} \leq \delta\beta_i^+ - \delta\beta_i^- \leq \frac{1}{\alpha_{iL}} - \frac{1}{\bar{\alpha}_i}$$

To maintain linear approximations, the design variable $\delta\beta_i$ is also bounded by a move limit M_i which is taken as a percent change from the current design point,

$$(62) \qquad |\delta\beta_i| \leq M_i$$

The bounds on the new design variables in equation (61) become,

$$(63) \qquad 0 \leq \delta\beta_i^+ \leq \text{MIN} \left| M_i, \left(\frac{1}{\alpha_{iL}} - \frac{1}{\bar{\alpha}_i}\right) \right|$$

$$0 \leq \delta\beta_i^- \leq \text{MIN} \left| M_i, \left(-\frac{1}{\alpha_{iu}} + \frac{1}{\bar{\alpha}_i}\right) \right|$$

The remainder of the optimization algorithm is converted by equation (60) to the $\delta\beta_i^+$, $\delta\beta_i^-$ space. For example, the acceleration constraint (54) in the time domain becomes,

$$\int_0^T I \quad dt = 0$$

where $$I = X_o + \Sigma \left(\frac{\partial \ddot{X}}{\partial \beta_i^+} \delta\beta_i^+ - \frac{\partial \ddot{X}}{\partial \beta_i^-} \delta\beta_i^- \right) - \beta, \qquad \text{for } \ddot{X} \geq \beta$$

$$I = -X_o - \Sigma \left(\frac{\partial \ddot{X}}{\partial \beta_i^+} \delta\beta_i^+ - \frac{\partial \ddot{X}}{\partial \beta_i^-} \delta\beta_i^- \right) - \beta \qquad -\ddot{X} \geq \beta$$

$$I = 0 \qquad \text{for } \ddot{X} \text{ otherwise}$$

The frequency constraint is changed similarly. At the end of each linear program, the new design variables are recovered by,

$$\beta_i = \bar{\beta}_i + \delta\beta_i^+ - \delta\beta_i^- \tag{64}$$

$$\alpha_i = \frac{1}{\beta_i}$$

11. Ill Conditioning

As mentioned previously, the vibration isolation problem can be ill conditioned. Isolators are designed to move independently of the main structure. This would require the properties to be several orders of magnitude less than the surrounding structure and conditioning problems would then arise. An approximate way of dealing with this situation which was done in reference (12) is to use modal synthesis. The main structure is modeled by component modes and the isolators are included at the syntheses time. If the generalized mass, stiffness and damping are of the same order of magnitude as the isolator properties, the problem will become well conditioned. However, this method is approximate and no error estimates are available for it.

Large space structures are very flexible and inherently ill conditioned. More accurate techniques would have to be used for problems in this category.

A method for solving ill conditioned eigensystems is presented in reference (16), but it may require modifications to be efficient for structural problems. A method for the solution of ill conditioned stiffness matrices is presented in reference (17) and it might be possible to modify it for the generalized eigenvalue problem. The numerical integration schemes for stiff equations are presented in reference (18).

12. Numerical Applications

Transient Response:

The model of reference (1) shown in figure 1 was subjected to the displacement inputs $f_1(t)$ and $f_2(t)$ shown in figure 2. This model represents a vehicle running over a bump. The transient step size used was .1 sec and 39 time intervals were calculated. The five springs were used as design variables with limits shown on figure 1. The objective function was a design variable which represented the maximum acceleration at point 1 in the model over the 3.9 sec time of response. The acceleration constraints were made active when the acceleration bound was 99% of the maximum. Figure 3 presents the decrease in acceleration at point 1 in the model versus the required number of structural analyses. By equation (60),each design variable is converted into two so the total number of design variables used in the linear program was eleven. Initially, the reciprocal variables were constrained by a move limit to lie within $\pm$25% of the initial values. Convergence was obtained at iteration three. The spring rates found at the optimum were,

$$k_1 = 51.2 \text{ lb/in}$$

$$k_2 = 200.\text{lb/in}$$

$$k_3 = 200.\text{lb/in}$$

$$k_4 = 1600. \text{ lb/in}$$

$$k_5 = 1000. \text{ lb/in}$$

The minimum acceleration obtained was 228.8 in/sec^2. Figure 4 presents the initial response versus the optimal one.

An indeterminate beam composed of rods, shear panels, springs and concentrated masses is shown in figure 5. The two point masses were subjected to a harmonic force input of

$$f(t) = \begin{cases} 1000 \text{ SIN } (\pi t) & 0 \leq t \leq \text{sec} \\ 0 & 2 \text{ sec} < t \end{cases}$$

Eight design variables were used in the analysis, 3 rod areas at top and bottom and the two scalar springs. The two vertical rods inside the shear wall remained unchanged at the initial area of the assembly. The vertical displacement response of the top two response points were required to be less than, the 9th design variable. This problem validated the extension of the method to finite elements and multiple response points.

This problem is also statically indeterminate and the reciprocal design variables used in the analysis were only an approximation. A lumped mass model was used for the rods and shear panels. The shear panel used was the hybird one derived in reference (19). The damping used was C = .01 LB-SEC/IN and a scalar viscous damper of this magnitude to ground was applied at every degree of freedom. Figure 6 presents the decrease in the maximum displacement vs structural iteration. Convergence was achieved at iteration 3 when the maximum displacement went from an initial value of .0166 IN to .0082 IN at the third iteration. The rods at the optimum all reached the upper bound of 2 IN^2 and the scalar springs had the value of 51,200 LB/IN which were near the lower bound but not exactly equal to it.

Throughout the optimization, the symmetry in the problem was preserved. This problem could be ill conditioned when the spring rates were more than a power of ten smaller than the lower limit. Typically isolator elements are designed to vibrate independently from the surrounding structure. Vibration isolation is inherently ill conditioned for this reason, and this research has not considered the ill conditioned problem.

Frequency Response:

The model shown in figure 1 was subjected to equal inphase displacement inputs $f_1(t) = f_2(t) = 5 \cos \omega t$ at the tire. This would represent a vehicle on a shaker table. The five springs were used as design variables with the limits shown of figure 1. The acceleration amplitudes were evaluated between 5 RAD/SEC and 44 RAD/SEC in steps of 1 RAD/SEC as the driving frequencies. The objective function was the maximum steady state acceleration amplitude at point 1 over the range of driving frequencies. The acceleration constraints were made active when the acceleration was 99% of the maximum. Figure 7 presents the decrease in acceleration amplitude at point 1 in the model versus the required number of structural analyses. Initially, the reciprocal variables were constrained by a move limit to lie within $\pm$25% of the initial values. When a step was not minimizing, the percent move limit on the reciprocal variables was decreased by 50% and the linear program was resolved at the previous design point. Convergence was achieved at iteration 7 which was close to the value found at iteration 5.

The spring rates at the optimum were as follows:

k_1 = 52.9 LB/IN

k_2 = 231.2 LB/IN

k_3 = 215.1 LB/IN

k_4 = 1000.0 LB/IN

k_5 = 1311.0 LB/IN

The minimum acceleration amplitude was found to be 318.7 IN/SEC^2. Figure 8 compares the initial acceleration amplitude in the frequency domain vs the optimized response.

Stationary Random Response:

The model shown in figure 1 was subjected to a random displacement at the tire patches as discussed in reference (20) with parameters that correspond to a smooth highway. A Frequency Response solution is first completed with a unit harmonic displacement $e^{i\omega t}$ at the front tire and a unit displacement with phase lag $e^{i\cdot(\omega t-\beta)}$ at the rear tire with phase angle $\phi = \frac{L}{V}$. The spectral density of the output in terms of the spectral density of the input and transfer function is,

$$S_o(\omega) = |H(\omega)|^2 S_I(\omega).$$

The transfer function is determined by using the acceleration output of the frequency response solution due to the unit harmonic input.
The spectral density acceleration was evaluated between 5 RAD/SEC and 44 RAD/SEC in steps of 1 RAD/SEC. The objective function was the design variable representing the maximum acceleration spectral density.

The acceleration constraint was made active when it was 99% of the maximum. Figure 9 presents the decrease in the objective function versus the required number of structural analyses. The reciprocal spring rates were constrained by $\pm$25% of the current value as move limits. At the detection of each infeasibility, the move limit was reduced by 50% of the current percent.
Convergence was achieved at iteration 3.
The spring rates at the optimum were,

K_1 = 51.2 LB/IN

K_2 = 200. LB/IN

K_3 = 200. LB/IN

K_4 = 1000. LB/IN

K_5 = 2000. LB/IN

The minimum spectral density was 49.61 $(IN/SEC^2)^2/Hz$. The initial and optimized spectral densities are presented in figure 10.

Transient Response with Frequency Constraints:

The model shown in figure 1 was again used with the same constraints and loading condition. An additional constraint was added on the inverse frequency:

$$KY = \lambda^{*} MY$$

$$\frac{1}{8000}\left(\frac{sec}{rad}\right)^2 < \lambda^{*} < \frac{1}{25} \quad \left(\frac{sec}{rad}\right)^2$$

A 10% move limit was imposed initially on the design variables and each time an infeasibility was found, the move limit was decreased by 50%. The minimum acceleration found was 286.9 IN/SEC^2 after six structural analyses. The analysis was done in both the direct eigenvalue and inverse eigenvalue space with the same convergence properties. Figure 11 presents the decrease in acceleration vs structural analysis in the inverse eigenvalue constraint. Near convergence was achieved at four analyses, but no conclusions concerning the eigenvalue constraint could be inferred from this problem.

13. Conclusions

The vibration isolation problem can be efficiently analyzed expressing the displacement or acceleration constraints as a first order Taylor series expansion in reciprocal design variables. The cumulative constraint folds many discrete constraints into a single one.

The method for approximating the displacement or acceleration constraints is consistent with the current algorithms for weight minimization.

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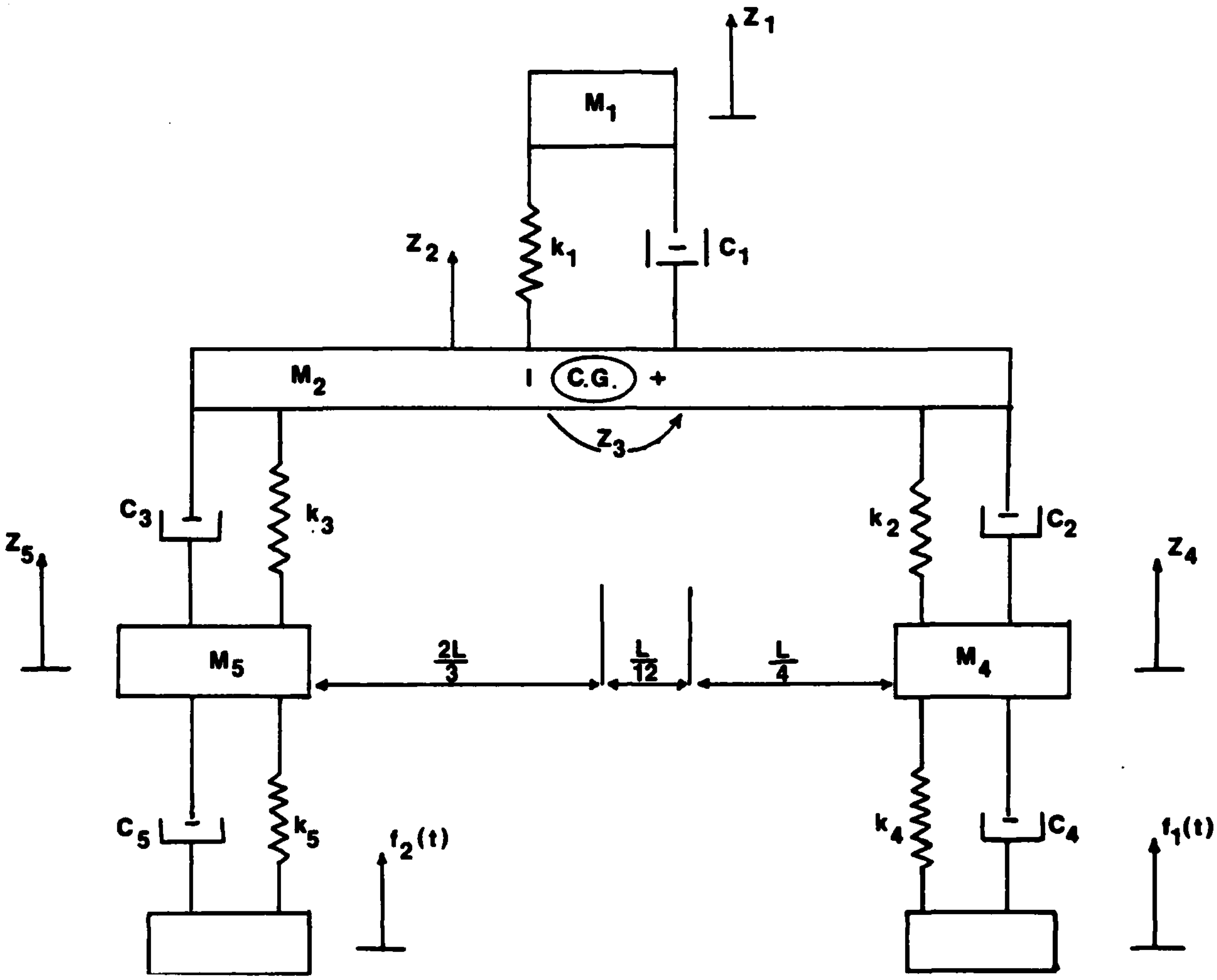
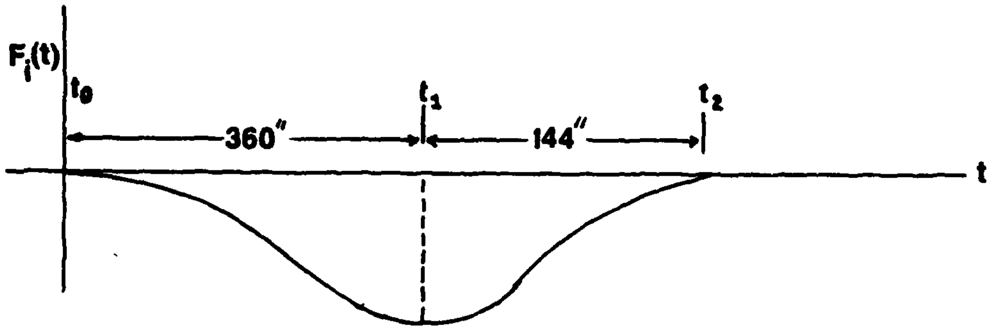


M_1G = 290 lb.
M_2G = 4,500 lb.
I = 41,000 lb.-in-sec
M_4G = 96.6 lb.
M_5G = 96.6 lb.

C_1 = 10 lb.-sec/in
C_2 = 25 lb.-sec/in
C_3 = 25 lb.-sec/in
C_4 = 5 lb.-sec/in
C_5 = 5 lb.-sec/in

LOWER LIMITS	INITIAL DESIGN	UPPER LIMITS
KL_1 = 50 lb./in	K_1 = 100 lb./in	KU_1 = 500 lb./in
KL_2 = 200 lb./in	K_2 = 300 lb./in	KU_2 = 1000 lb./in
KL_3 = 200 lb./in	K_3 = 300 lb./in	KU_3 = 1000 lb./in
KL_4 = 1000 lb./in	K_4 = 1500 lb./in	KU_4 = 2000 lb./in
KL_5 = 1000 lb./in	K_5 = 1500 lb./in	KU_5 = 2000 lb./in

FIGURE 1: OPTIMIZATION MODEL



AMPLITUDE $X_0 = 5''$

VEHICLE SPEED $S = 450$ IN/SEC

WHEEL BASE $L = 120$ IN

$d_1 = 360''$, $d_2 = 144''$

$w_1 = \frac{\pi S}{d_1} = 1.25\pi$

$w_2 = \frac{\pi S}{d_2} = 3.125\pi$

$t_1 = d_1/_S = .8$ SEC, $t_2 = (d_1+d_2)/_S = 1.12$ SEC

TIME LAG FRONT TO REAR t_L

$t_L = L/_S = .2667$ SEC

FRONT WHEEL DISPLACEMENT

$f_1(t) = X_0(1 - \cos wt) \qquad 0 \le t \le t_1$

$f_1(t) = X_0(1 + \cos w(t - t_1)) \qquad t_1 \le t \le t_2$

REAR WHEEL DISPLACEMENT

$f_2(t) = f_1(t - t_2) \qquad 0 \le t - t_L \le t_2$

FIGURE 2: TRANSIENT DISPLACEMENT INPUT FOR FIGURE 1

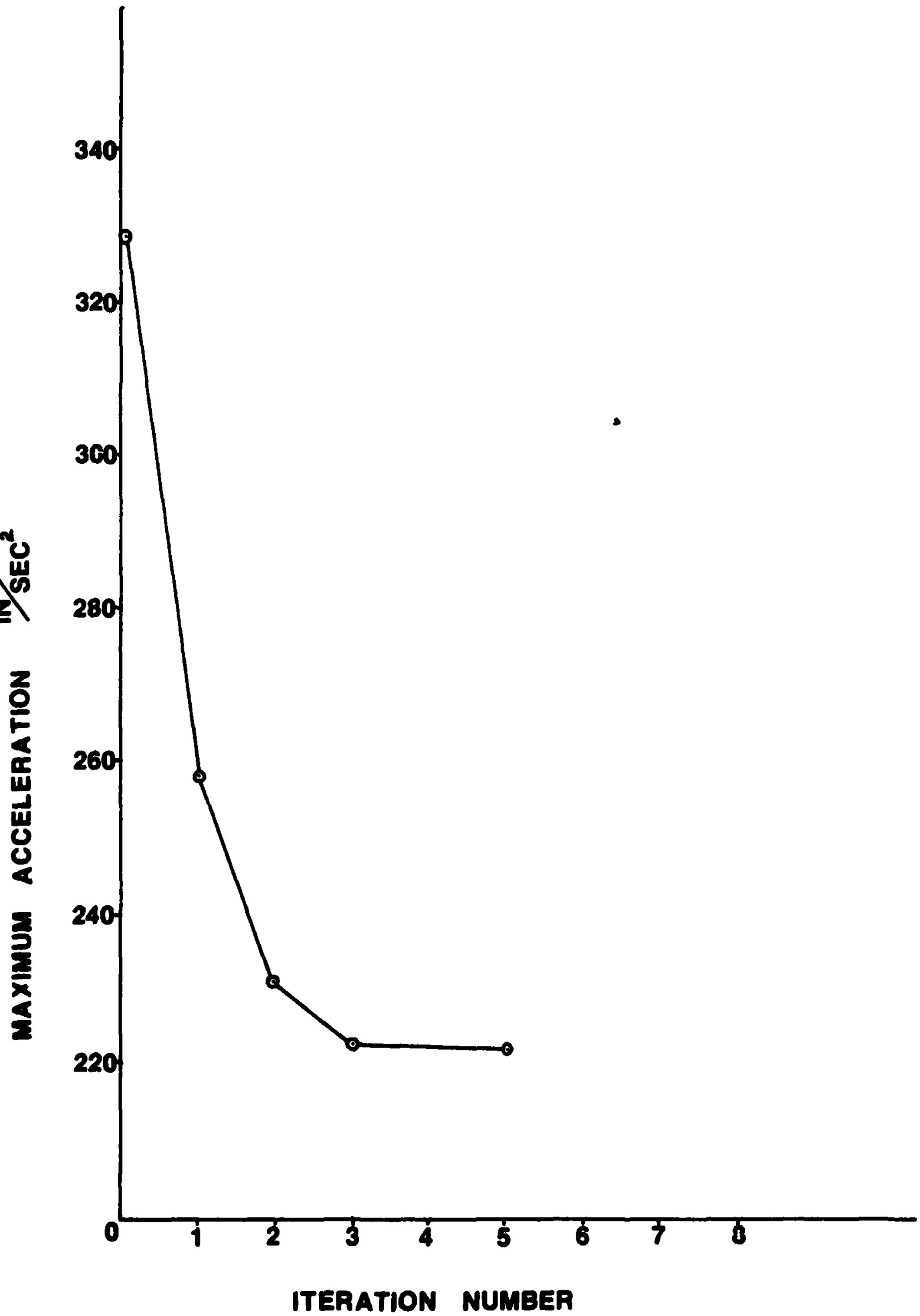


FIGURE 3: TRANSIENT RESPONSE OF FIGURE 1

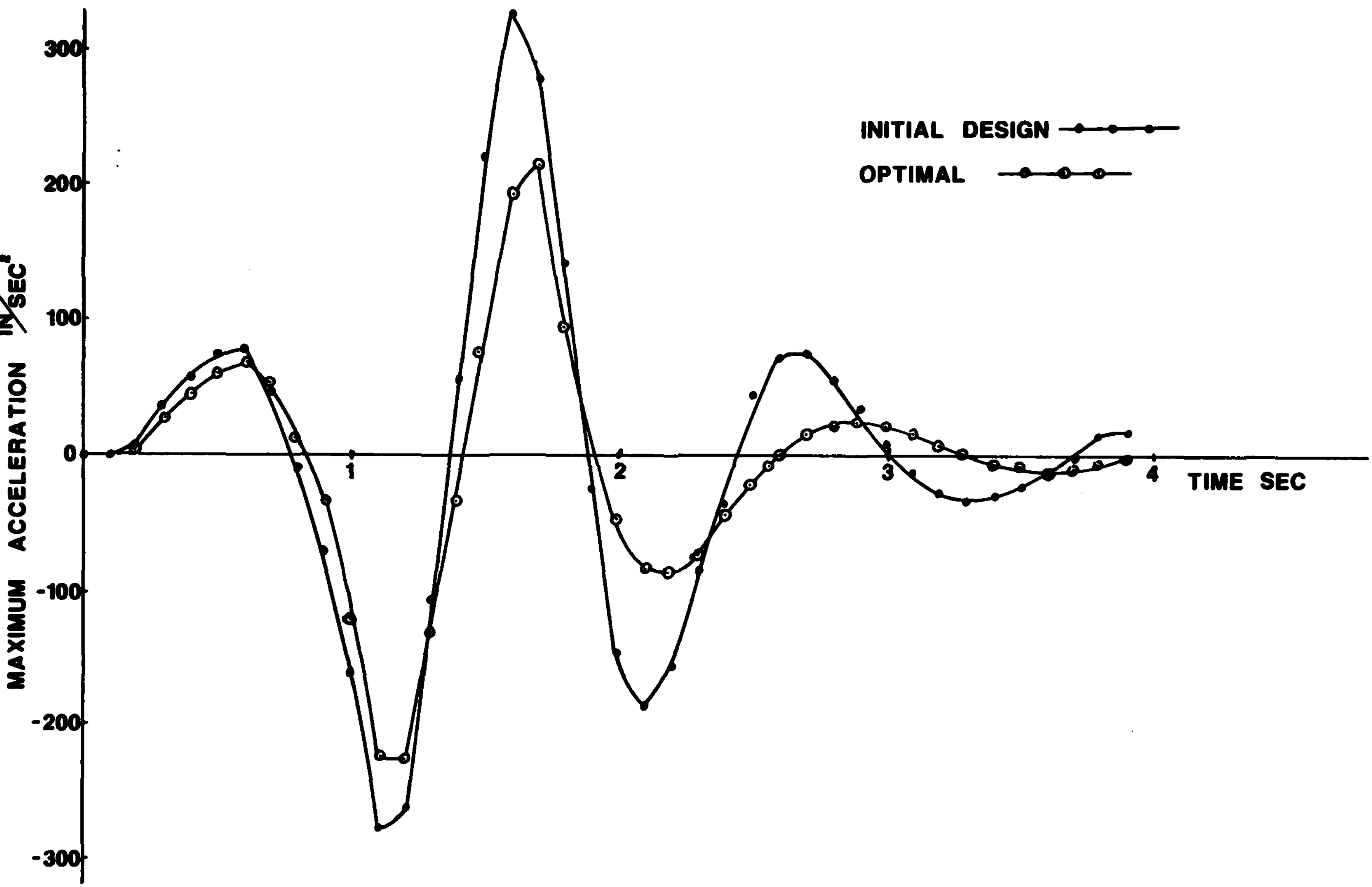
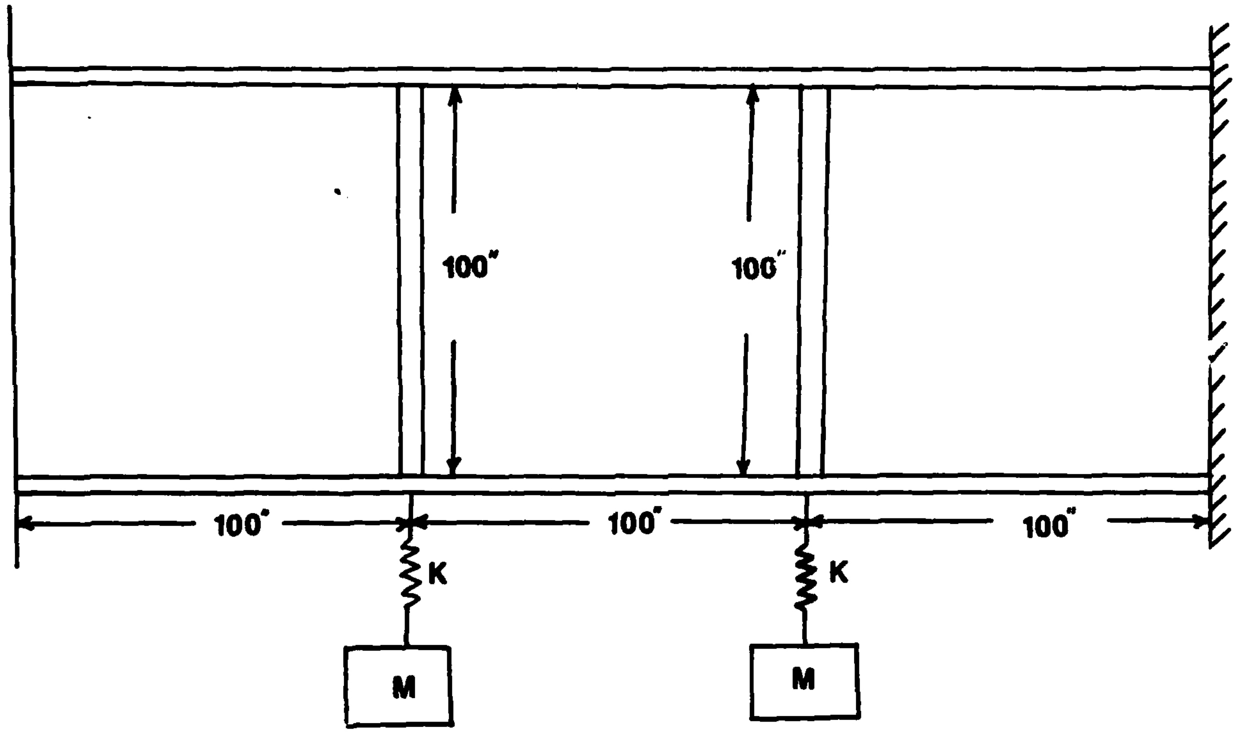


FIGURE 4: TRANSIENT RESPONSE OF FIGURE 1



LOWER LIMIT	INITIAL AREAS	UPPER LIMIT
.5 IN^2	1 IN^2	2 IN^2

$E = 10 \times 10^6$ PSI

$G = 4 \times 10^6$ PSI

$\rho = 8.79 \times 10^{-3}$ SLUGS/IN^3

$t =$ .1 IN

M = .005 SLUGS

C = .01 LB-SEC/IN

LOWER LIMIT	INITIAL K	UPPER LIMIT
50,000 LB/IN.	100,000 LB/IN.	200,000 LB/IN.

FIGURE 5: RODS, SHEAR PANELS, SPRINGS AND MASSES

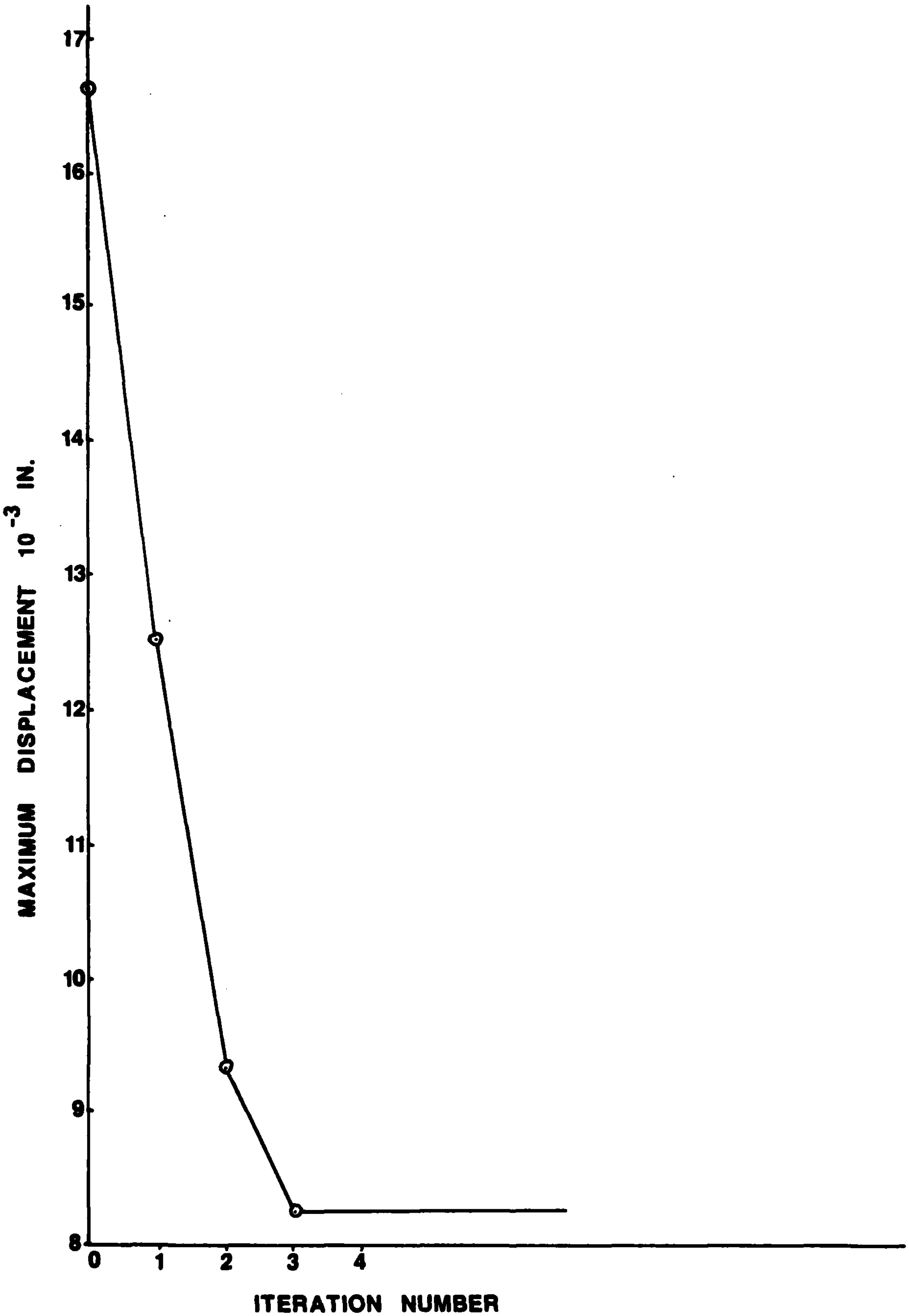


FIGURE 6: TRANSIENT RESPONSE OF FIGURE 5

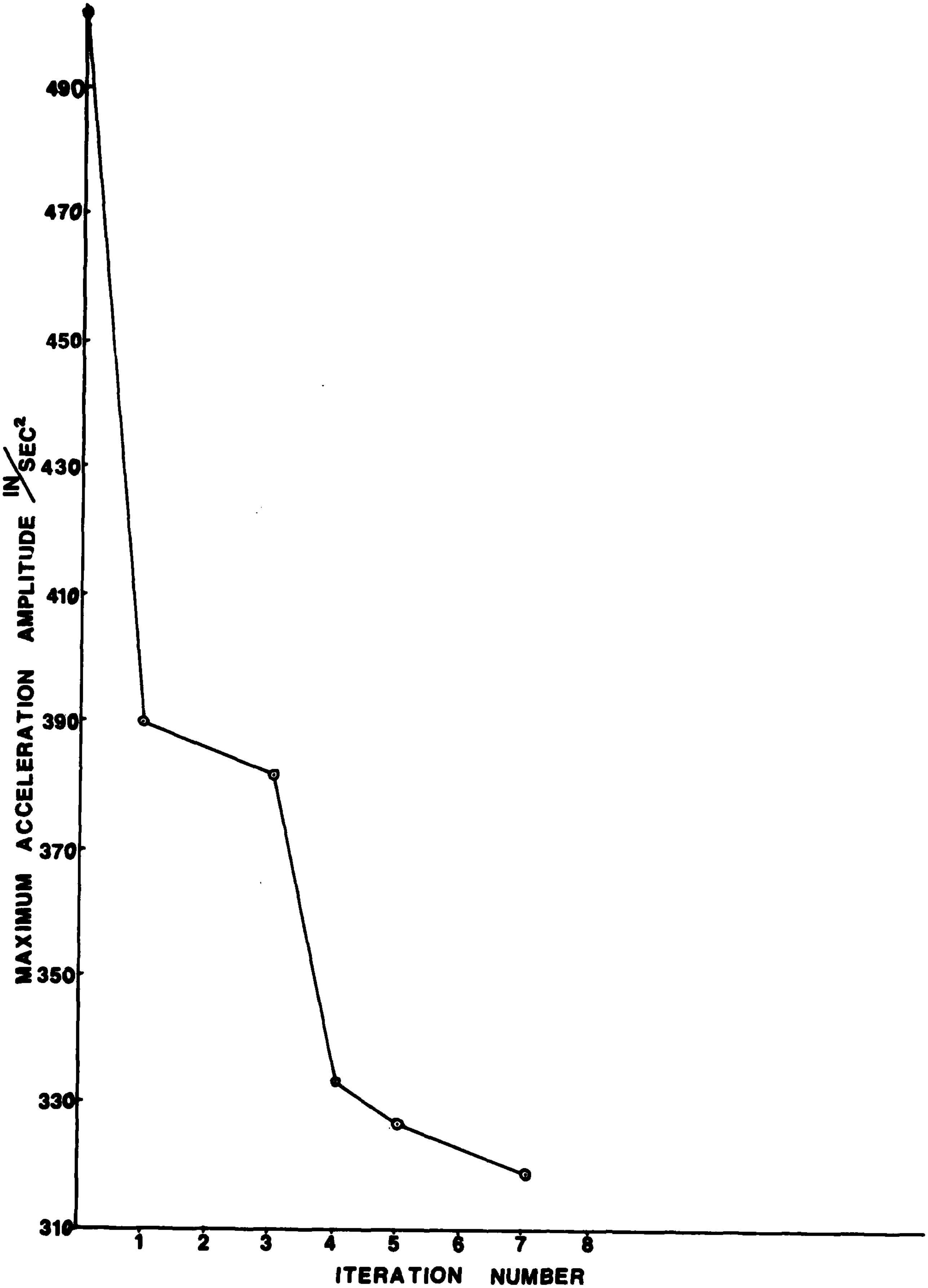


FIGURE 7: FREQUENCY RESPONSE OF FIGURE 1

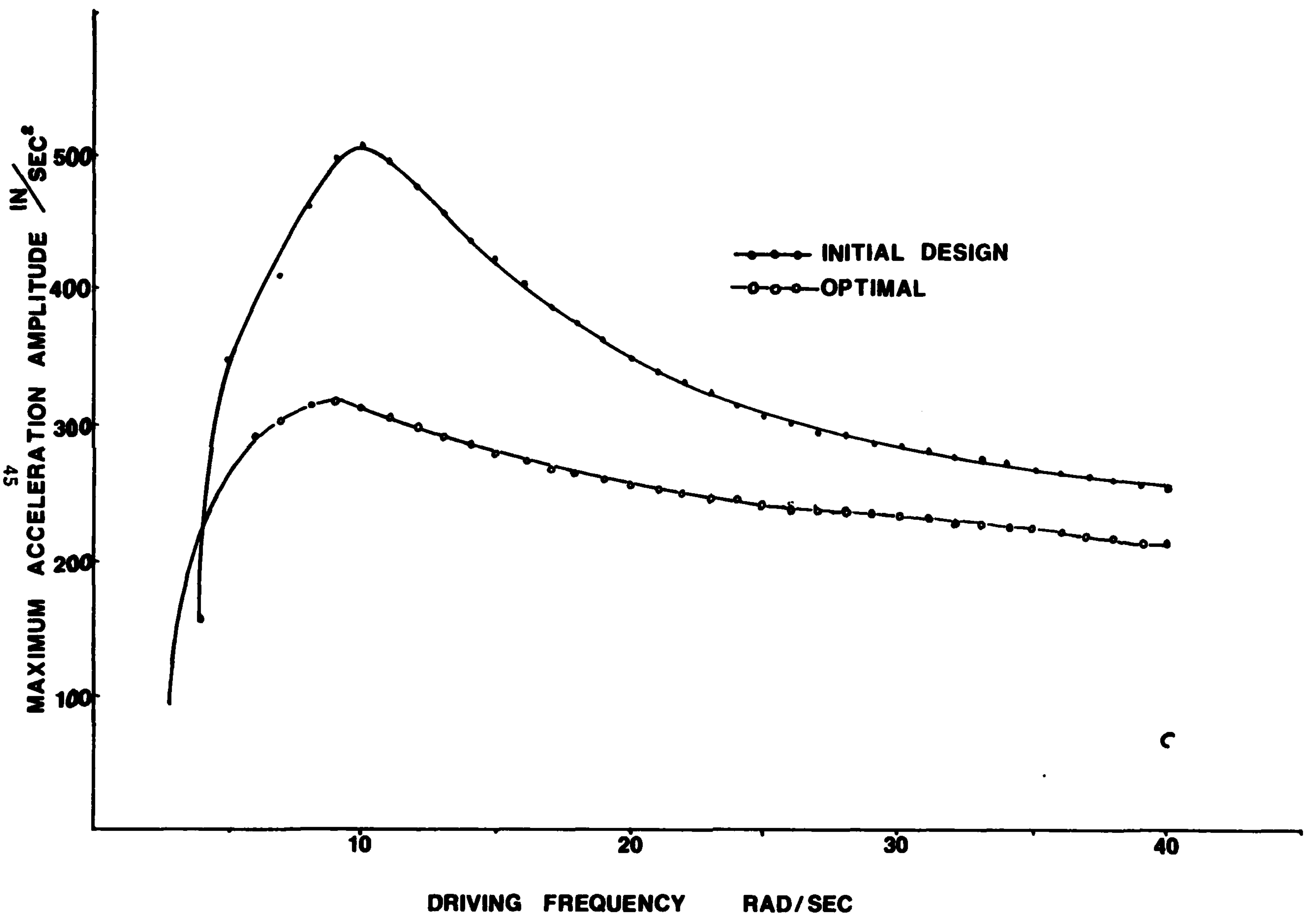


FIGURE 8: FREQUENCY RESPONSE OF FIGURE 1

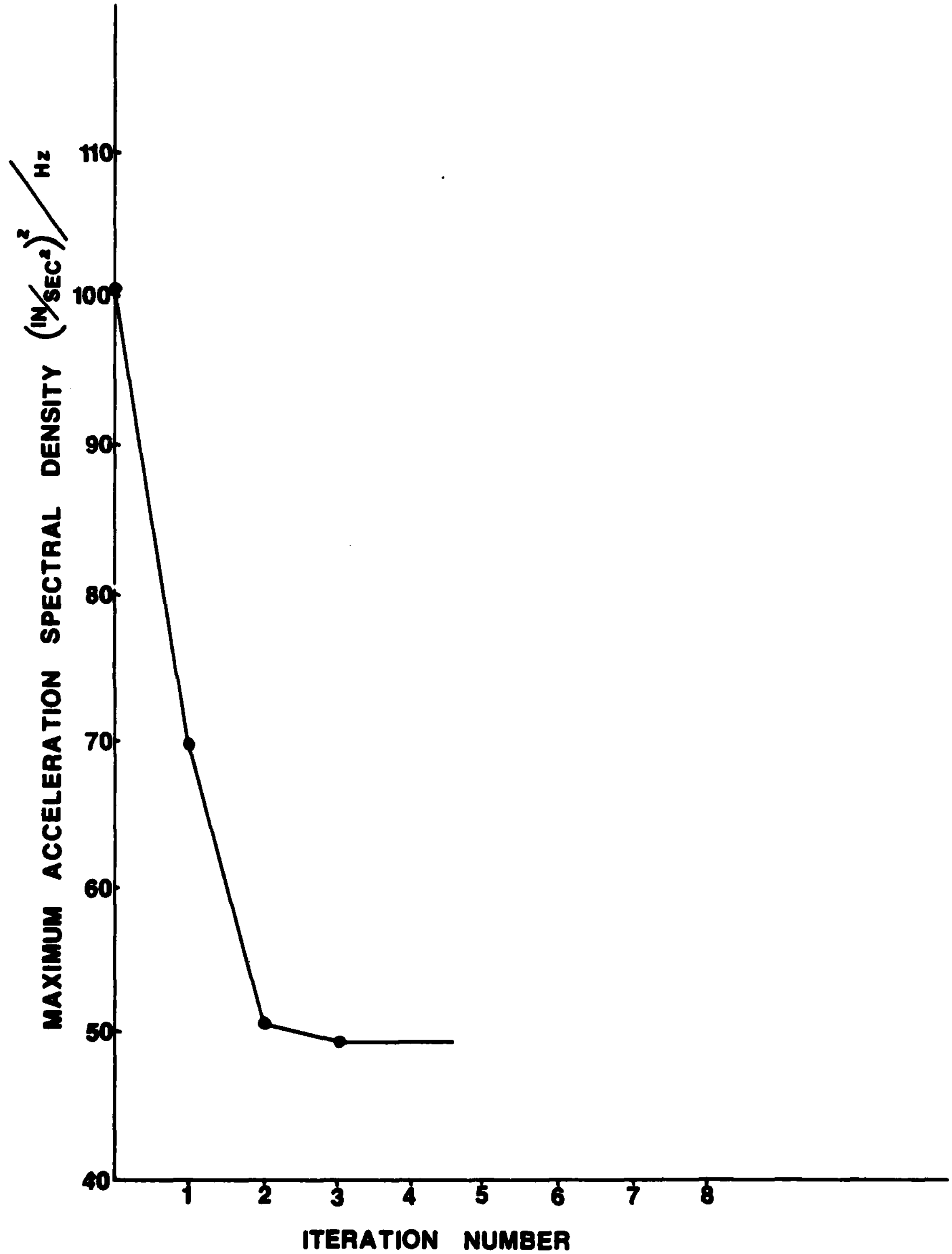


FIGURE 9: STATIONARY RANDOM RESPONSE OF FIGURE 1

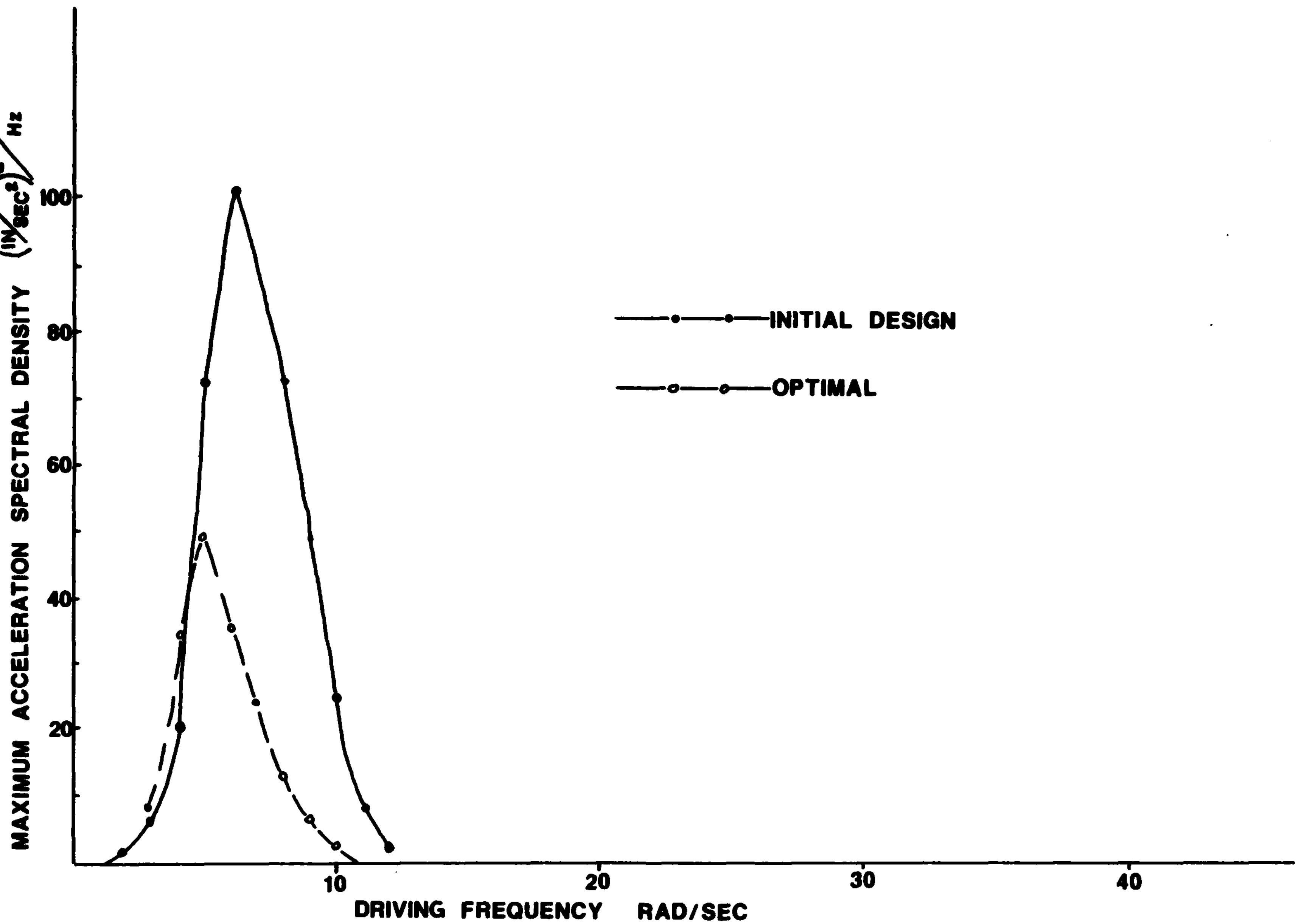


FIGURE 10: STATIONARY RANDOM RESPONSE OF FIGURE 1

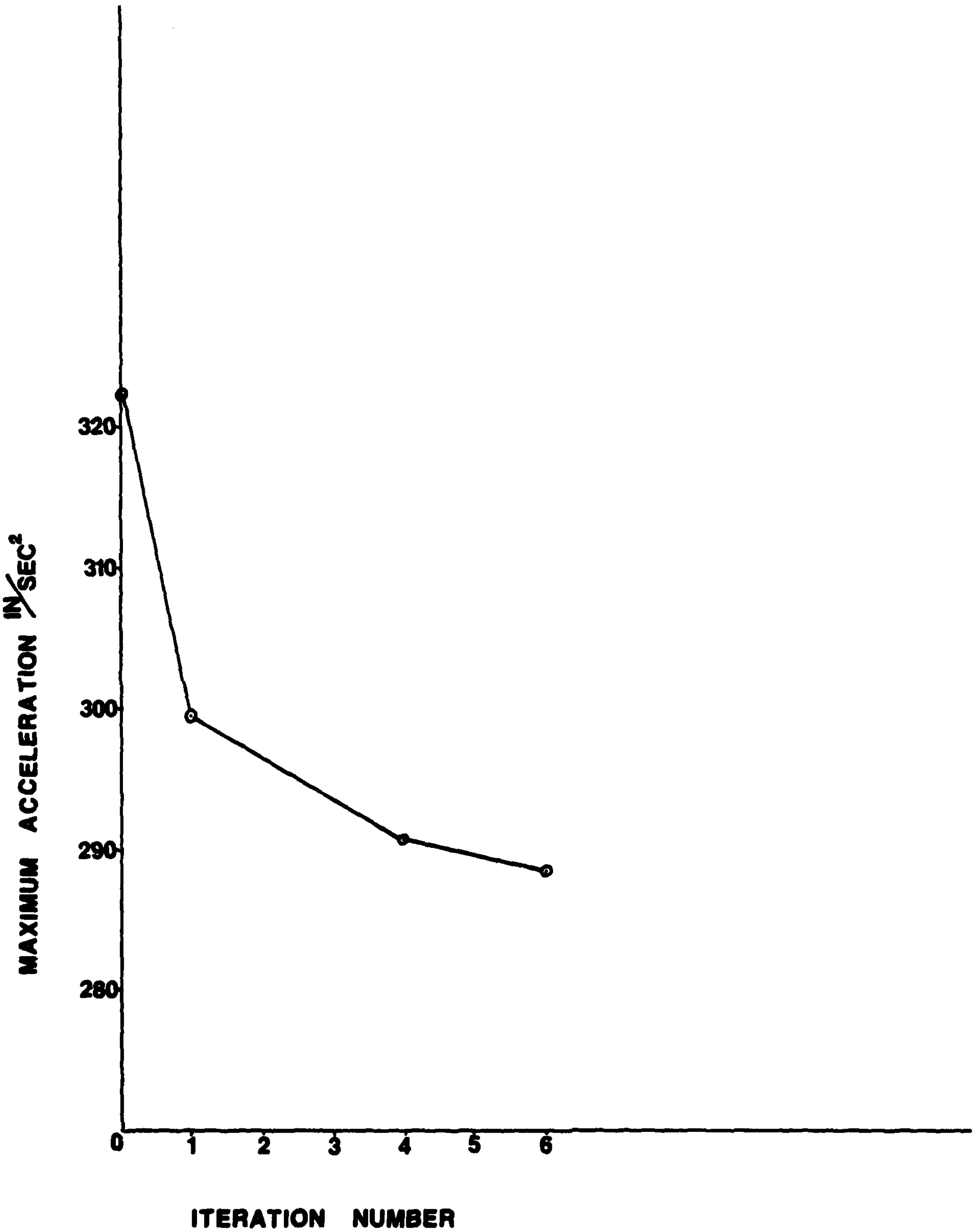


FIGURE 11: TRANSIENT RESPONSE WITH FREQUENCY CONSTRAINT OF FIGURE 1

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